System temperatures of 21 cm arrays 2015

D. C. Jacobs

with significant contribution from:
A. Beardsley, B. Hazelton, N. Thyagarajan, and C. Trott

November 5, 2015

Here we report system temperatures of some 21 cm reionization arrays and assemble a handy list of references to past measurements and theoretical predictions. The goal here is to get at relative amplitude between estimates, so we'll stick to a single frequency of 180MHz and an LST of 0 hours.

Most of these temperatures are derived by differencing visibilities and then converting back to an apparent sky temperature. See Section 1 for a laboriously exacting definition of the relationship between sky temperature, antenna temperature and visibility rms.

Table 1: Temperatures at 180MHz and LST=0h

Instrument/method	Predicted $T_{sky}[K]^8$	Design $T_{rcvr}[K]$	Predicted T_{sys}	Measured T_{sys} [K]	Measured T_{sky} [K]
MWA (RTS to CHIPS)	180	30[1]	210	212.5	
MWA (RTS to EPPSILON)	180	30[1]	210	236[2]	
MWA (FHD to EPPSILON)	180	30[1]	210	230	
PAPER	200[3]	100	300	376[4]	
EDGES	180	40	220	NA	180.1
LOFAR	317[6]	??	>317	??	??
HERA	180	TBD	TBD	TBD	

[1] uncertain to 10 to 20K
[2] measured 190K at 196MHz
[3] slightly higher due to wider PAPER beam, a bit more uncertain than MWA
[4] measured 355K at 175MHz, published number (480K) scaled up by 1.35 correction factor, which was incorrectly calculated and has since been revised to 1.06
[6] (Labropoulos et al., 2009) 60 @ 300MHz scaled to 180 and up by sqrt(2) to account for their equation is only Tsys of the real part.
[8] Calculated by N. Thyagarajan

Table 2: Some Tsys citations

Measurement of MWA Tsys	Bowman et al. (2007)
quotes T_{rcvr} for MWA	Tingay et al. (2013)
The "Briggs" rule for T_{sky} (180K at 180 MHz)	Furlanetto et al. (2006)
the de Oliviera-Costa model	de Oliveira-Costa et al. (2008)
Measurement of PAPER Tsys	Ali et al. (2015)
Measurement of EDGES Tsys and list of older measurements	Rogers and Bowman (2008)
The LOFAR Tsys (arxiv, not yet published)	Labropoulos et al. (2009)

1 A laboriously exacting definition of system temperature

Here is how we get from our visibilities to a system temperature.

The system temperature of a radio interferometer comes from two components, T_{sky} and T_{rcvr}. Let T_{sky} be defined as the average of the sky temperature, weighted by the primary beam.

$$T_{sky} = \frac{\int \text{Beam}(\theta,\phi) T_{\text{sky map}}(\theta,\phi)}{\int \text{Beam}(\theta,\phi)} \tag{1}$$

Where the sky map is the de Oliviera Costa compilation of diffuse measurements evaluated at 180MHz and the Beam here is the sum the two beams for two polarizations. All calculations in this memo assume a polarization definition where stokes I is the sum of the two linear components.

$$I = XX + YY \tag{2}$$

Thus the temperature calculated in a single polarization will be half the temperature in the stokes I.

The de Oliviera Costa model also predicts T_{sky} to have a power law slope of -2.55 which was confirmed by Rogers and Bowman (2009). Measurements from 85MHz to 408MHz have been found to extrapolate well to 180MHz.

The measured system temperatures reported here are calculated by differencing between two sets of visibilities, integrated in opposing even/odd cadences

$$\sigma_{vis,xx,imag} = \sqrt{\frac{\sum_{ijt}\left(\mathbb{R}(V)^{\text{even}}_{ijt} - \mathbb{R}(V)^{\text{odd}}_{ijt}\right)^2}{2(N-1)}} \tag{3}$$

This is usually done in the real or imaginary plane of the visibilities. To get what the amplitude variance would be we scale up by $\sqrt{2}$

$$\sigma_{vis,xx} = \sigma_{vis,imag}\sqrt{2} \tag{4}$$

So, the $\sqrt{2}$ cancels the $\sqrt{2}$ but its totally an accident

$$\sigma_{vis,xx} = \sqrt{\frac{\sum_{ijt}\left(\mathbb{R}(V)^{\text{even}}_{ijt} - \mathbb{R}(V)^{\text{odd}}_{ijt}\right)^2}{N-1}} \tag{5}$$

Thus our visibility variance is estimated. Now to convert to a temperature.

1.1 Temperature on a single polarization

The temperature induced in a single polarization (say xx) antenna is, according to the 1D Raleigh-Jeans law,

$$P_{xx,ant} = k_B T_{xx,ant} \Delta\nu \tag{6}$$

meanwhile the power captured from a polarized source S_{xx} with a collecting area A is

$$P_{xx,ant} = S_{xx} A \Delta\nu \tag{7}$$

Combining the two gives me the relationship between the x half of the flux and the temperature measured on the output.

$$T_{xx,ant} = \frac{S_{xx} A}{k_B} \tag{8}$$

Note the complete absence of '2's.

1.2 Sky temperature

So far we have only computed the "antenna temperature", to get a "sky" temperature we have to invoke the 3D Raleigh-Jeans law relating the temperature and emissivity

$$I(\nu) = \frac{2\nu^2 k_B T_{sky}}{c^2} \tag{9}$$

This has units of Watts / m^2 / str / Hz. To relate to the power received by a radio element we multiply by the collecting area A, the angle subtended by the element $d\Omega$ and a bandwidth $\Delta\nu$ and most importantly, the antenna is by definition a single polarization element so we divide by 2[1]

$$P_{xx,ant} = \frac{2\nu^2 k_B T_{sky} A d\Omega \Delta\nu}{2c^2} \tag{10}$$

Setting equation 10 equal to equation 6 we find that

$$T_{sky} = T_{xx,ant} \tag{11}$$

Thus the 2 from 3D Raleigh-Jeans conspires with our use of a single dipole antenna 2 to make the temperature we read out of a calorimeter attached to our single polarization antenna (T_{ant}) equal to the stokes I temperature of the sky T_{sky}.

1.3 sky temperature from visibility rms

If the sky temperature is equal to the single polarization temperature we need only look to equation 8 for our conversion to temperature and substitute equation 5 for the S_{xx}.

S_{xx}, the instantaneous variance is related to the observed variance by the integration time dt and frequency resolution df

$$S_{xx,noise} = \sigma_{vis,xx}\sqrt{df\,dt} \tag{12}$$

$$T_{sky} = T_{xx,ant} = \frac{A\sigma_{vis,xx}\sqrt{df\,dt}}{k_B} \tag{13}$$

References

Z. S. Ali, A. R. Parsons, H. Zheng, J. C. Pober, A. Liu, J. E. Aguirre, R. F. Bradley, G. Bernardi, C. L. Carilli, C. Cheng, D. R. DeBoer, M. R. Dexter, J. Grobbelaar, J. Horrell, D. C. Jacobs, P. Klima, D. H. E. MacMahon, M. Maree, D. F. Moore, N. Razavi, I. I. Stefan, W. P. Walbrugh, and A. Walker. PAPER-64 Constraints on Reionization: The 21 cm Power Spectrum at z = 8.4. ApJ, 809:61, Aug. 2015. doi: 10.1088/0004-637X/809/1/61.

J. Bowman, D. Barnes, F. Briggs, and B. Corey. Field deployment of prototype antenna tiles for the mileura widefield array low frequency .... *The Astronomical Journal*, Jan 2007. URL `http://www.iop.org/EJ/article/1538-3881/133/4/1505/205565.web.pdf`.

A. de Oliveira-Costa, M. Tegmark, B. M. Gaensler, J. Jonas, T. L. Landecker, and P. Reich. A model of diffuse galactic radio emission from 10 mhz to 100 ghz. *Monthly Notices of the Royal Astronomical Society*, 388:247, Jun 2008. doi: 10.1111/j.1365-2966.2008.13376.x. URL `http://adsabs.harvard.edu/cgi-bin/nph-data_query?bibcode=2008MNRAS.388..247D&link_type=ABSTRACT`. (c) Journal compilation © 2008 RAS.

S. R. Furlanetto, S. P. Oh, and F. H. Briggs. Cosmology at low frequencies: The 21 cm transition and the high-redshift universe. *Physics Reports*, 433:181, Sep 2006. doi: 10.1016/j.physrep.2006.08.002. URL `http://adsabs.harvard.edu/cgi-bin/nph-data_query?bibcode=2006PhR...433..181F&link_type=ABSTRACT`. Elsevier B.V.

[1] note that we are assuming that the sky is unpolarized on the scale probed by the antenna element

P. Labropoulos, L. V. E. Koopmans, V. Jelic, S. Yatawatta, R. M. Thomas, G. Bernardi, M. Brentjens, G. de Bruyn, B. Ciardi, G. Harker, A. Offringa, V. N. Pandey, J. Schaye, and S. Zaroubi. The lofar eor data model: (i) effects of noise and instrumental corruptions on the 21-cm reionization signal-extraction strategy. *eprint arXiv*, 0901:3359, Jan 2009. URL `http://adsabs.harvard.edu/cgi-bin/nph-data_query?bibcode=2009arXiv0901.3359L&link_type=ABSTRACT`. 20 pages, 9 figures.

A. E. E. Rogers and J. D. Bowman. Spectral Index of the Diffuse Radio Background Measured from 100 to 200 MHz. *Astronomical Journal*, 136:641–648, Aug. 2008. doi: 10.1088/0004-6256/136/2/641.

S. Tingay, R. Goeke, J. Bowman, D. Emrich, S. Ord, D. Mitchell, M. Morales, T. Booler, B. Crosse, R. Wayth, C. Lonsdale, S. Tremblay, D. Pallot, T. Colegate, A. Wicenec, N. Kudryavtseva, W. Arcus, D. Barnes, G. Bernardi, F. Briggs, S. Burns, J. Bunton, R. Cappallo, B. Corey, A. Deshpande, L. Desouza, B. Gaensler, L. Greenhill, P. Hall, B. Hazelton, D. Herne, J. Hewitt, M. Johnston-Hollitt, D. Kaplan, J. Kasper, B. Kincaid, R. Koenig, E. Kratzenberg, M. Lynch, B. Mckinley, S. Mcwhirter, E. Morgan, D. Oberoi, J. Pathikulangara, T. Prabu, R. Remillard, A. Rogers, A. Roshi, J. Salah, R. Sault, N. Udaya-Shankar, F. Schlagenhaufer, K. Srivani, J. Stevens, R. Subrahmanyan, M. Waterson, R. Webster, A. Whitney, A. Williams, C. Williams, and J. Wyithe. The murchison widefield array: The square kilometre array precursor at low radio frequencies. *Publications of the Astronomical Society of Australia*, 30:7–7, Jan 2013. doi: doi:10.1017/pasa.2012.007. URL `http://adsabs.harvard.edu/abs/2013PASA...30....7T`.